# INDIANA CORE ASSESSMENTS FOR EDUCATOR LICENSURE

## FIELD 070: MIDDLE SCHOOL MATHEMATICS

## TEST FRAMEWORK

## September 2020

| Domain | | Range of Objectives | Approximate Percentage of Test Score |
|---|---|---|---|
| I. | Number Sense and Computation | 0001–0002 | 24% |
| II. | Algebra and Functions | 0003–0006 | 34% |
| III. | Measurement and Geometry | 0007–0009 | 14% |
| IV. | Probability, Statistics, and Calculus | 0010–0012 | 14% |
| V. | Instruction and Assessment | 0013 | 14% |

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# TEST FRAMEWORK

Number Sense and Computation
Algebra and Functions
Measurement and Geometry
Probability, Statistics, and Calculus
Instruction and Assessment

## NUMBER SENSE AND COMPUTATION

### 0001 Understand the structure and properties of number systems.

For example:

- Recognize the hierarchy of the real and complex number systems, their classification into various subsets, and properties of number systems (e.g., commutative, distributive, associative, inverse and identity elements, closure).
- Reason about the ordering and absolute value of real numbers.
- Apply and extend understanding of the place value system to represent, estimate, and perform operations on rational numbers in a variety of forms (e.g., graphic, numerical, physical, symbolic).
- Justify and apply order of operations to real and complex numbers.
- Apply and extend understanding of prime and composite numbers, divisibility, least common multiples, and greatest common factors to model and solve mathematical and real-world problems.
- Apply and analyze standard and nonstandard algorithms for operations on decimals, fractions, integers, and other real numbers.
- Model and solve mathematical and real-world problems using scientific notation and the properties of integer exponents.
- Apply basic properties of matrices, matrix arithmetic, vectors, and vector arithmetic.

**0002 Understand rational numbers, ratios, and proportional relationships.**

For example:

- Use benchmark numbers, number sense, properties, and rounding to estimate mentally and assess the reasonableness of solutions to mathematical and real-world problems.
- Solve mathematical and real-world problems involving positive and negative rational numbers using a variety of methods.
- Use equations, area models, tiles, diagrams, and other visual and physical models to represent fractions and to perform arithmetic operations on fractions.
- Use visual models and strategies based on place value and properties of operations to solve problems involving conversions between fractions, percentages, repeating decimals, and terminating decimals.
- Apply proportionality, rates, ratios, and unit rates to solve problems involving discounts and markups, interest, percent increase and decrease, taxes, tips, and other real-world situations.

## ALGEBRA AND FUNCTIONS

**0003 Understand algebraic expressions, equations, and inequalities.**

For example:

- Translate between algebraic, graphic, numerical, symbolic, tabular, and verbal descriptions that represent mathematical and real-world situations.
- Evaluate absolute value, exponential, linear, quadratic, rational, and square root algebraic expressions for a given value of a variable and express one variable in terms of another variable.
- Manipulate and simplify absolute value, exponential, linear, quadratic, rational, and square root algebraic expressions and solve related equations and inequalities.
- Use algebraic expressions, equations, and inequalities with rational coefficients to model and solve mathematical and real-world problems.
- Solve algebraic equations and inequalities having one, multiple, infinitely many, extraneous, or no solutions.
- Justify algebraic techniques using the properties of real numbers.

## 0004 Understand algebraic relations and functions.

For example:

- Use algebraic, graphic, tabular, and verbal representations to describe relationships between quantities and to analyze and distinguish between relations and functions.
- Use patterns, relations, sequences, and series (e.g., Fibonacci, arithmetic, geometric) to model and solve mathematical and real-world problems.
- Generate and interpret equations, graphs, tables, and other representations of real-world situations and translate between them.
- Analyze, describe, and solve linear and nonlinear mathematical and real-world situations using addition, subtraction, multiplication, division, and composition of functions.
- Identify the effects of transformations such as $f(x + k)$, $f(x) + k$, and $k(f(x))$ on the graphs of linear and nonlinear functions, where $k$ is a real number.

## 0005 Understand linear relations and functions.

For example:

- Analyze connections between direct variation, constants of proportionality, linear models, proportional relationships, and rates of change and use these connections to build linear functions.
- Analyze the relationship between the equation of a line and its graph and interpret slope and intercepts in mathematical and real-world situations.
- Determine the equation of a line in slope-intercept, standard, and point-slope forms from various types of information (e.g., graphs, one point and slope, two points).
- Use elimination, graphing, and substitution for solving systems of linear equations and inequalities.
- Apply knowledge of linear equations, functions, inequalities, and systems and slope of a line to analyze, model, and solve real-world problems and situations.

**0006 Understand nonlinear relations and functions.**

For example:

- Identify, express, and apply patterns of change created by exponential, inversely proportional, and quadratic situations.
- Translate between algebraic, graphic, tabular, and verbal descriptions of exponential, inversely proportional, and quadratic functions.
- Analyze absolute value, exponential, logarithmic, polynomial, rational, and trigonometric functions and relations with respect to continuity, domain, end behavior, extrema, inverse, range, and other characteristics.
- Use completing the square, the discriminant, factoring, graphing, and the quadratic formula to model and solve problems involving quadratic relations and functions (e.g., conic sections).
- Use exponential functions to model and solve problems involving population growth, compound interest, half-life, and other exponential growth and decay situations.

## MEASUREMENT AND GEOMETRY

**0007 Understand units and measurement.**

For example:

- Extend knowledge of quantities and units to compare and convert within and between measurement systems and use these conversions in solving real-world problems.
- Solve problems involving derived units.
- Analyze the effect of measurement error and rounding on computed quantities.
- Solve mathematical and real-world problems using angles, area, length, perimeter, surface area, and volume to find measures of two- and three-dimensional figures and composite shapes.
- Analyze the effect of changing linear dimensions on measures of length, area, and volume.
- Use concepts of congruence, indirect measurement, proportional and spatial reasoning, and similarity to analyze and solve mathematical and real-world problems.
- Apply the Pythagorean theorem, right triangle trigonometry, and special right triangle relationships to solve mathematical and real-world problems.
- Apply periodic phenomena and trigonometric identities.

**0008 Understand Euclidean geometry.**

For example:

- Use properties and theorems about angle pairs, parallel lines, and perpendicular lines to characterize geometric relationships and solve problems.
- Use properties of sides, angles, and diagonals to analyze and justify relationships between triangles, quadrilaterals, and other polygons and use these relationships to solve mathematical and real-world problems.
- Use properties and theorems about circles to solve mathematical and real-world problems involving arc lengths, tangents, angle measures, and other circle measurements.
- Use knowledge of the axiomatic structure of Euclidean geometry to analyze and prove theorems.
- Analyze and compare the properties and measurements of circular cones, circular cylinders, hemispheres, prisms, pyramids, and spheres.

**0009 Understand Cartesian coordinate and transformational geometry.**

For example:

- Use concepts of distance, midpoint, slope, and parallel and perpendicular lines to classify, represent, and analyze triangles, quadrilaterals, circles, and other geometric figures in the coordinate plane.
- Analyze dilations, translations, rotations, and reflections of figures in two-dimensional coordinate space.
- Analyze the effects of transformations on figures with respect to congruence, similarity, and symmetry.
- Use coordinate and transformational geometry to evaluate logical arguments and mathematical conjectures and to prove theorems and solve problems.
- Use coordinate geometry techniques to model and solve mathematical and real-world problems involving two- and three-dimensional figures.

## PROBABILITY, STATISTICS, AND CALCULUS

### 0010 Understand probability.

For example:

- Use and interpret organized lists, sample spaces, tables, tree diagrams, Venn diagrams, and other representations for situations involving probability.
- Compute theoretical probabilities for simple and compound events using addition rules, multiplication rules, and other approaches.
- Select simulations to generate frequencies and experimental probabilities for simple and compound events.
- Use combinations and permutations to represent and solve mathematical and real-world problems involving probability.

### 0011 Understand statistics.

For example:

- Construct and interpret scatter plots to investigate patterns of association between two quantities, interpret and estimate correlation coefficients, and solve problems involving linear regression models.
- Construct and interpret frequency distributions, tables, bar charts, circle graphs, dot plots, stem-and-leaf plots, box plots, and histograms.
- Describe and summarize numerical data sets by identifying clusters, modes (e.g., peaks), gaps, and symmetry and by considering the context in which the data were collected.
- Use mean, median, interquartile range, and standard deviation to make comparisons.
- Evaluate experiments, observational studies, and sampling procedures in terms of bias, randomization, sample size, and other methods for gathering and organizing data.

**0012 Understand calculus.**

For example:

- Apply the concepts of limits and continuity to analyze functions and their graphs.
- Use the relationships between derivatives, slopes, and rates of change to model and solve mathematical and real-world problems.
- Use derivatives to find maxima, minima, points of inflection, and concavity of curves.
- Find antiderivatives, evaluate integrals, and apply the Fundamental Theorem of Calculus to find the area under a curve.
- Apply the concepts of differentiation and integration to model and solve mathematical and real-world problems.

## INSTRUCTION AND ASSESSMENT

**0013 Understand content and process standards and instructional strategies.**

For example:

- Identify and analyze appropriate mathematics instructional strategies and skills using state and national standards.
- Support the learning of all students by using materials, pacing, student interest, student engagement, and methods for differentiating instruction to make mathematics content accessible.
- Apply a variety of strategies and communication methods that promote critical thinking; foster inquiry, interaction, and collaboration; and improve, deepen, and broaden understanding of the mathematics curriculum.
- Use standards-based instruction and assessment to develop and evaluate middle school mathematics lessons.
- Select and create high-quality curricula and materials that match students' needs and educational goals.
- Use formal and informal assessment to plan, implement, and evaluate effective instruction.
- Select and integrate appropriate technology to engage students in solving authentic problems and to facilitate learning, creativity, and innovation.